The symmetry, connecting the processes in 2- and 4-dimensional space-times, and the value $\alpha_0 = 1/4\pi$ for the bare fine structure constant

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Abstract

The symmetry manifests itself in exact mathematical relations between the Bogoliubov coefficients for the processes induced by accelerated point mirror in 1+1-space and the current (charge) densities for the processes caused by accelerated point charge in 3+1-space. The spectrum of pairs of Bose (Fermi) massless quanta, emitted by mirror, coincides with the spectrum of photons (scalar quanta), emitted by electric (scalar) charge up to multiplier $e^2/\hbar c$. The integral connection of the propagator of a pair of oppositely directed massless particles in 1+1-space with the propagator of a single particle in 3+1-space leads to equality of the vacuum-vacuum amplitudes for charge and mirror if the mean number of created particles is small and the charge $e = \sqrt{\hbar c}$. Due to the symmetry the mass shifts of electric and scalar charges, the sources of Bose-fields with spin 1 and 0 in 3+1-space, for the trajectories with subluminal relative velocity β_{12} of the ends and maximum proper acceleration w_0 are expressed in terms of heat capacity (or energy) spectral densities of Bose- and Fermi-gases of massless particles with temperature $w_0/2\pi$ in 1+1-space. Thus, the acceleration excites the one-dimensional oscillations in the proper field of a charges and the energy of oscillations is partly deexcited in the form of real quanta and partly remains in the field. As a result, the mass shift of accelerated electric charge is nonzero and negative , while that of scalar charge is zero. The symmetry is extended to the mirror and charge interactions with the fields carrying spacelike momenta and defining the Bogoliubov coefficients $\alpha^{B,F}$. The traces $\mathrm{tr}\,\alpha^{B,F}$ describe the vector and scalar interactions of accelerated mirror with a uniformly moving detector and were found in analytical form for two mirror's trajectories with subluminal velocities of the ends. The symmetry predicts one and the same value $e_0 = \sqrt{\hbar c}$ for electric and scalar charges in 3+1-space. The arguments are adduced in favour of that this value and the corresponding value $\alpha_0 = 1/4\pi$ for fine structure constant are the bare, nonrenormalized values.

1 Introduction

The Hawking's mechanism for particle production at the black hole formation is analogous to the emission from an ideal mirror accelerated in vacuum [1]. In its turn there is a close analogy between the radiation of pairs of scalar (spinor) quanta from accelerated mirror in 1+1 space and the radiation of photons (scalar quanta) by an accelerated electric (scalar) charge in 3+1 space [2,3]. Thus all these processes turn out to be mutually related. In problems with moving mirrors the *in*-set $\phi_{in\,\omega'}$, $\phi^*_{in\,\omega'}$ and *out*-set $\phi_{out\,\omega}$, $\phi^*_{out\,\omega}$ of the wave

equation solutions are usually used. For massless scalar field they look as follows:

$$\phi_{in\,\omega'}(u,v)=\frac{1}{\sqrt{2\omega'}}[e^{-i\omega' v}-e^{-i\omega' f(u)}],\qquad \phi_{out\,\omega}(u,v)=\frac{1}{\sqrt{2\omega}}[e^{-i\omega g(v)}-e^{-i\omega u}],\qquad (1)$$

with zero boundary condition $\phi|_{traj}=0$ on the mirror's trajectory. Here the variables $u=t-x$, $v=t+x$ are used and the mirror's (or charge's) trajectory on the u, v plane is given by any of the two mutually inverse functions $v^{mir}=f(u)$, $u^{mir}=g(v)$.

For the *in-* and *out*-sets of massless Dirac equation solutions see [3]. Dirac solutions differ from (1) by the presence of bispinor coefficients at u- and v-plane waves. The current densities corresponding to these solutions have only tangential components on the boundary. So, the boundary condition both for scalar and spinor field is purely geometrical, it does not contain any dimensional parameters.

The Bogoliubov coefficients $\alpha_{\omega'\omega}$, $\beta_{\omega'\,\omega}$ appear as the coefficients of the expansion of the *out*-set solutions in the *in*-set solutions; the coefficients $\alpha^*_{\omega'\omega}$, $\mp\beta_{\omega'\omega}$ arise as the coefficients of the inverse expansion. The upper and lower signs correspond to scalar (Bose) and spinor (Fermi) field. The explicit form of Bogoliubov coefficients is very simple:

$$\alpha^B_{\omega'\omega},\;\beta^{B*}_{\omega'\omega}=\sqrt{\frac{\omega'}{\omega}}\int_{-\infty}^{\infty}dv\,e^{i\omega' v\mp i\omega g(v)}=\pm\sqrt{\frac{\omega}{\omega'}}\int_{-\infty}^{\infty}du\,e^{\mp i\omega u+i\omega' f(u)}.\qquad (2)$$

The coefficients $\alpha^F_{\omega'\omega}$, $\beta^{F*}_{\omega'\omega}$ for Fermi-field differ from these representations by the changes $\sqrt{\omega'/\omega}\to\sqrt{g'(v)}$, $\pm\sqrt{\omega/\omega'}\to\sqrt{f'(u)}$ under the integral signs.

Then the mean number $d\bar{n}_\omega$ of quanta radiated by accelerated mirror to the right semi-space with frequency ω and wave vector $\omega>0$, and the total mean number $\bar{N}$ of quanta are given by the integrals

$$d\bar{n}^{B,F}_\omega=\frac{d\omega}{2\pi}\int_0^\infty\frac{d\omega'}{2\pi}|\beta^{B,F}_{\omega'\omega}|^2,\qquad \bar{N}^{B,F}=\iint\limits_0^\infty\frac{d\omega d\omega'}{(2\pi)^2}|\beta^{B,F}_{\omega'\omega}|^2.\qquad (3)$$

These expressions do not contain $\hbar$, but their interpretation as mean numbers of quanta follows from the second-quantized theory. The second-quantized theory allows to construct the all possible amplitudes of many-particle creation, annihilation and scattering via Bogoliubov coefficients [4,5,6].

At the same time the spectra of photons and scalar quanta emitted by electric and scalar charges moving along the trajectory $x_\alpha(\tau)$ in 3+1 space are defined by the Fourier transforms of the electric current density 4-vector $j_\alpha(x)$ and the scalar charge density $\rho(x)$,

$$j_\alpha(k),\,\rho(k)=e\int d\tau\,\{\dot{x}_\alpha(\tau),\,1\}e^{-ik^\alpha x_\alpha(\tau)},\quad j_\alpha(x),\,\rho(x)=e\int d\tau\,\{\dot{x}_\alpha(\tau),\,1\}\delta_4(x-x(\tau)),\qquad (4)$$

and are given by the formulae

$$d\bar{n}^{(1,0)}_k=\frac{1}{\hbar c}\{|j_\alpha(k)|^2,\;|\rho(k)|^2\}\frac{dk_+dk_-}{(4\pi)^2},\qquad \bar{N}^{(1,0)}=\frac{1}{\hbar c}\iint\limits_0^\infty\frac{dk_+dk_-}{(4\pi)^2}\{|j_\alpha(k)|^2,\;|\rho(k)|^2\},\quad (5)$$

where the upper index in $d\bar{n}_k^{(s)}$, $\bar{N}^{(s)}$, and k^α denote the spin and 4-momentum of quanta,

$$k^2 = k_1^2 + k_\perp^2 - k_0^2 = 0, \quad k_\perp^2 = k_0^2 - k_1^2 = k_+k_-, \quad k_\pm = k^0 \pm k^1,$$

and in (5) it is supposed that the trajectory $x^\alpha(\tau)$ has only x^0 and x^1 nontrivial components, as the mirror's one.

In contrast to quantities in (3), the $d\bar{n}_k^{(s)}$ and $\bar{N}^{(s)}$ in (5) contain $\hbar$ since the charge entering into current and charge densities is considered as classical quantity. In essence, the $d\bar{n}_k^{(s)}$ and $\bar{N}^{(s)}$ can be considered as classical quantities because they are obtained from purely classical radiation energy spectrum $d\bar{\mathcal{E}}_k^{(s)}$ divided by the energy $\hbar k^0$ of single quantum, so that

$$d\bar{n}_k^{(s)} = \frac{d\bar{\mathcal{E}}_k^{(s)}}{\hbar k^0}, \quad \bar{N}^{(s)} = \int \frac{d\bar{\mathcal{E}}_k^{(s)}}{\hbar k^0}, \quad k^0 = \frac{1}{2}(k_+ + k_-). \tag{6}$$

The symmetry between the creation of Bose or Fermi pairs by accelerated mirror in 1+1 space and the emission of single photons or scalar quanta by electric or scalar charge in 3+1 space consists, first of all, in the coincidence of the spectra. If one puts $2\omega = k_+$, $2\omega' = k_-$, then

$$|\beta^B_{\omega'\omega}|^2 = \frac{1}{e^2}|j_\alpha(k_+, k_-)|^2, \qquad |\beta^F_{\omega'\omega}|^2 = \frac{1}{e^2}|\rho(k_+, k_-)|^2. \tag{7}$$

So, the spectra coincide as a functions of two variables and a functionals of common trajectory of a mirror and a charge. The distinction in multiplier $e^2/\hbar c$ can be removed if one puts $e^2 = \hbar c$.

The symmetry under discussion connecting the classical and quantum theories in Minkowsky spaces of 4 and 2 dimensions in some sense reminds the duality of classical and quantum descriptions in spaces of neighbour dimensions which was proposed by G.'t Hooft [7] and L. Susskind [8]. Such a duality really discovered by Gubser, Klebanov and Polyakov [9] and by Maldacena [10] for different types of quasiclassical supergravity in anti de Sitter space and quantum conformal theories on the boundary of this space. It seems plausible that the general reason of such a dualities consists in the correspondence between a single particle in space of larger dimension and a pair of particles in space of smaller dimension. The description of a larger number of particles in space of smaller dimension needs in accounting the quantum mechanical interference effects.

2 Symmetry and physical content and distinction of $\beta^*_{\omega'\omega}$ and $\alpha_{\omega'\omega}$

It follows from the second-quantized theory that the absolute pair production amplitude and the single-particle scattering amplitude are connected by the relation

$$\langle \text{out}\omega''\omega|\text{in}\rangle = -\sum_{\omega'} \langle \text{out}\omega''|\omega'\text{in}\rangle\, \beta^*_{\omega'\omega}. \tag{8}$$

It enables to interpret $\beta^*_{\omega'\omega}$ as the amplitude of a source of a pair of the massless particles potentially emitted to the right and to the left with frequences ω and ω' respectively [6]. While the particle with frequency ω actually escapes to the right, the particle with frequency

ω' propagates some time then is reflected by the mirror and is actually emitted to the right with altered frequency ω''. Then, in the time interval between pair creation and reflection of the left particle, we have the virtual pair with energy k^0, momentum k^1, and mass m:

$$k^0 = \omega + \omega', \quad k^1 = \omega - \omega', \quad m = \sqrt{-k^2} = 2\sqrt{\omega\omega'}. \tag{9}$$

Apart from this polar timelike 2-vector k^α, very important is the axial spacelike 2-vector q^α,

$$q_\alpha = \varepsilon_{\alpha\beta}k^\beta, \quad q^0 = -k^1 = -\omega + \omega', \quad q^1 = -k^0 = -\omega - \omega' < 0. \tag{10}$$

With the help of k^α and q^α the symmetry between α and β coefficients becomes clearly expressed:

$$s = 1, \qquad e\beta^{B*}_{\omega'\omega} = -\frac{q_\alpha j^\alpha(k)}{\sqrt{k_+k_-}}, \qquad e\alpha^B_{\omega'\omega} = -\frac{k_\alpha j^\alpha(q)}{\sqrt{k_+k_-}}, \tag{11}$$

$$s = 0, \qquad e\beta^{F*}_{\omega'\omega} = \rho(k), \qquad e\alpha^F_{\omega'\omega} = \rho(q). \tag{12}$$

Note that the equations (4) define the current density $j^\alpha(k)$ and the charge density $\rho(k)$ as the functionals of the trajectory $x^\alpha(\tau)$ and the functions of any 2- or 4-vector k^α. It can be shown that in 1+1-space $j^\alpha(k)$ and $j^\alpha(q)$ are the spacelike and timelike polar vectors if k^α and q^α are the timelike and spacelike vectors correspondingly.

The boundary condition on the mirror evokes in the vacuum of massless scalar or spinor field the appearance of vector or scalar disturbance waves bilinear in massless fields. There are two types of these waves:

1) The waves with amplitude $\alpha_{\omega'\omega}$ ($\alpha^*_{\omega'\omega}$) which carry the spacelike momentum directed to the left (right), and

2) The waves with amplitude $\beta^*_{\omega'\omega}$ ($\beta_{\omega'\omega}$) which carry the timelike momentum with positive (negative) frequency.

The waves with the spacelike momenta appear even if the mirror is in rest or moves uniformly (Casimir effect), while the waves with the timelike momenta appear only in the case of accelerated mirror.

The pair of Bose (Fermi) particles has spin 1 (0) because its source is the current density vector (charge density scalar), see [11] or the problem 12.15 in [12].

3 Vacuum-vacuum amplitude $\langle \mathrm{out}|\mathrm{in}\rangle = e^{iW}$, self-action and mass shifts

It follows from the secondary quantized theory that in the vacuum-vacuum amplitude $\langle \mathrm{out}|\mathrm{in}\rangle = e^{iW}$ the $\mathrm{Im}\, W^{B,F}$ is well-defined. According to DeWitt [4], Wald [5] and others (including myself [6])

$$2\,\mathrm{Im}\, W^{B,F} = \pm\frac{1}{2}\mathrm{tr}\, \ln(1 \pm \beta^+\beta) \quad \text{or} \quad \pm\, \mathrm{tr}\, \ln(1 \pm \beta^+\beta) \tag{13}$$

correspondingly to the cases when particle is identical or nonidentical to antiparticle. We confine ourselves by the last case and by the smallness of the $\mathrm{tr}\, \beta^+\beta \ll 1$. Then

$$2\,\mathrm{Im}\, W^{B,F} \approx \mathrm{tr}\, (\beta^+\beta)^{B,F} \equiv \iint\limits_0^\infty \frac{d\omega d\omega'}{(2\pi)^2}\, |\beta^{B,F}_{\omega'\omega}|^2 = \bar{N}^{B,F}. \tag{14}$$

By using in the integrand of $\bar{N}^{B,F}$ the representations (2) for $\beta^{B,F}$, the variables $x_{\mp}(\tau)$ and $x_{\pm}(\tau')$ instead of u, $f(u)$ and v, $g(v)$, and hyperbolic variables ρ, ϑ instead of ω, ω',

$$d\omega d\omega' = \frac{1}{2}\rho d\rho d\vartheta, \quad \omega = \frac{1}{2}\rho e^{\vartheta}, \quad \omega' = \frac{1}{2}\rho e^{-\vartheta}, \quad \rho = 2\sqrt{\omega\omega'}, \quad \vartheta = \ln\sqrt{\frac{\omega}{\omega'}}, \tag{15}$$

one obtains the imaginary part of the causal function in 1+1-space, $\mathrm{Im}\,\Delta_2^f(z,\rho)$, after integration over ϑ, and then the imaginary part of the causal function in 3+1-space, $\mathrm{Im}\,\Delta_4^f(z,\mu)$, after integration over $\rho = m$, the variable which coincides with the mass of virtual pair according to (9). This result is a special case of the very important integral relation between the causal functions of wave equations for d- and $d+2$-dimensional space-times [13],

$$\Delta_{d+2}^f(z,\mu) = \frac{1}{4\pi}\int_{\mu^2}^{\infty} dm^2\, \Delta_d^f(z,m), \tag{16}$$

The small mass parameter $\mu = 2\sqrt{\omega\omega'}|_{min} \neq 0$ is introduced instead of zero to avoid the infrared divergency in the following. Thus we obtain

$$2\,\mathrm{Im}\,W^{B,F} = \mathrm{Im}\iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^f(z,\mu), \qquad z_\alpha = x_\alpha(\tau) - x_\alpha(\tau'). \tag{17}$$

We may omit the Im-signs from both of sides of this equation and define the actions for Bose- and Fermi-mirrors in 1+1-space as

$$W^{B,F} = \frac{1}{2}\iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^f(z,\mu). \tag{18}$$

Compare this with the well known actions for electric and scalar charges in 3+1-space:

$$W_{1,0} = \frac{1}{2}e^2\iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^f(z,\mu). \tag{19}$$

The symmetry would be complete if $e^2 = 1$, i.e. if the fine structure constant were $\alpha = 1/4\pi$. This "ideal" value of fine structure constant for the charges would correspond to the ideal, geometrical boundary condition on the mirror.

The appearance in action the causal function $\Delta_4^f(z,\mu)$ has a lucid physical grounds.

1. The action must represent not only the radiation of real quanta but also the self-energy and polarization effects. While the first effects are described by the solutions of homogeneous wave equation the second ones require the inhomogeneous wave equation solutions which contain information about proper field of a source. Namely such solutions of homogeneous and inhomogeneous wave equations are the functions $(1/2)\Delta^1 = \mathrm{Im}\,\Delta^f$ and $\bar{\Delta} = \mathrm{Re}\,\Delta^f$.

2. While the appearance of $\mathrm{Im}\,\Delta^f$ in the imaginary part of the action (17) is a consequence of mathematical transformation of the integral $\bar{N}^{B,F}$ (similar to the Plancherel theorem), the function $\bar{\Delta} \equiv \mathrm{Re}\,\Delta^f$ in the real part of the action is unique if it appears as the real part of the analytical continuation of the function $i\,\mathrm{Im}\,\Delta^f(z,\mu)$ to negative z^2 that is even in z as $\mathrm{Im}\,\Delta^f$ itself.

Both the propagator $\Delta_2^f(z,m)$ of a virtual pair with mass $m=\rho=2\sqrt{\omega\omega'}$ in two-dimensional space-time and the mass spectrum of these pairs arise owing to the transition from the variables ω, ω' to the hyperbolic variables ρ, ϑ, which reflect the Lorentz symmetry of the problem. Further integration over the mass leads to the propagator $\Delta_4^f(z,\mu)$ of a particle moving in four-dimensional space-time with the mass μ equal to the least mass of virtual pairs. Thus, the relation (16) is immanent to the Lorentz symmetry and the symmetry, connecting the processes in two- and four-dimensional space-times.

For the point-like charges the $W_{1,0}$ contain ultraviolet divergences and need in their elimination. The removal of ultraviolet divergences in the self-actions $W_{1,0}|^F$ of accelerated charges (force $F\neq 0$) consists in the subtraction of corresponding self-actions $W_{1,0}|^{F=0}$ of uniformly moving charges as a result of which the changes

$$\varDelta W_{1,0}=W_{1,0}|_0^F=W_{1,0}|^F-W_{1,0}|^{F=0}$$

of self-actions owing to acceleration do not contain ultraviolet singularities, have the positive imaginary part, $\mathrm{Im}\,\varDelta W_{1,0}>0$, and vanish together with acceleration.

The following representations for the self-actions of uniformly moving electric and scalar charges are very instructive

$$W_{1,0}|^{F=0}=\frac{1}{2}\,e^2\iint d\tau\,d\tau'\{\dot x_\alpha(\tau)\dot x^\alpha(\tau'),\ 1\}\,\Delta_4^f(z,\mu)|^{F=0}=\mp\frac{e^2}{4\pi}\cdot\frac{1-i}{2\sqrt{2\varepsilon}}\cdot\tau. \qquad (20)$$

They arise if one introduces the integration variable $x=\tau'-\tau$ instead of τ', so that $z^2=-x^2$, puts $\mu=0$, and makes use of representation

$$\Delta_4^f(z,\mu)|_{\mu=0}=-\frac{1}{4\pi^2}\cdot\frac{i}{x^2-i\varepsilon}=\frac{1}{4\pi^2}(\frac{\varepsilon}{x^4+\varepsilon^2}-i\frac{x^2}{x^4+\varepsilon^2}),\quad \varepsilon\to 0.$$

The opposite signs of the self-actions are due to repulsion of like electric charges and to attraction of scalar ones. The coefficients before τ are the classical proper energies $-\delta m_{1,0}$ of the charges taken with minus sign, and $\sqrt{2\varepsilon}$ characterizes the charge dimension. Different signs of $\mathrm{Im}\,W_{1,0}|^{F=0}$ lead, according to amplitudes $\exp(iW_{1,0}|^{F=0})$, to disappearance (screening) of electric charge and to unlimited growing (antiscreening) of scalar charge.

These extraordinary properties of the self-actions arise from the pointlikeness of the charges. For the vector- and scalar-field sources $j^\alpha(x)$ and $\rho(x)$ distributed in space and slow varying in time the self-actions are free from singularities and have no imaginary parts [11]:

$$W_{1,0}=\int dt\int\frac{d^3x\,d^3x'}{4\pi|\mathbf{x}-\mathbf{x}'|}\{j_\alpha(x)\,j^\alpha(x'),\quad \rho(x)\,\rho(x')\}_{t'=t}. \qquad (21)$$

In this form self-actions contain the Ampere's and Coulomb laws for current and charge interactions and the law of attraction of like scalar charges. Self-actions (20), (21) are in accordance with general assertion that the interaction of like charges transfered by odd-spin quanta leads to repulsion while by even-spin quanta - to attraction.

We exemplify here the self-action changes $\varDelta W_{1,0}$ of electric and scalar charges due to accelerated motion along the very important quasihyperbolic trajectory

$$x(t)=\frac{\beta_1^2}{w_0}-\beta_1\sqrt{\frac{\beta_1^2}{w_0^2}+t^2},\quad \beta_{1,2}=\pm\mathrm{th}\,\frac{\theta}{2},\quad \beta_{12}=\frac{\beta_1-\beta_2}{1-\beta_1\beta_2}=\mathrm{th}\,\theta, \qquad (22)$$

with initial β_1 and final β_2 velocities at $t = \mp\infty$ and proper acceleration $-w_0$ at $t = 0$. The proper acceleration at any moment is given by the formula

$$a(t) = -\frac{w_0}{(1 + t^2/t_c^2)^{3/2}}, \quad t_c = \frac{\beta_1}{w_0\sqrt{1-\beta_1^2}}. \tag{23}$$

Therefore, the quasihyperbolic motion is close to hyperbolic one on the time interval $|t| < t_c$.

The self-action changes $\mathit{\Delta} W_{1,0}(\theta, \lambda)$ are the Lorentz invariant functions of two variables $\theta = \mathrm{Arth}\,\beta_{12}$ and $\lambda = \mu^2/w_0^2$ with singularities at $\lambda = 0$ and $\theta = \pm\infty$.

The case $\lambda \to 0,\ \theta$ arbitrary, was considered by author in [13]:

$$\mathit{\Delta} W_1 = \frac{e^2}{8\pi^2}\{\pi(\frac{\theta}{\mathrm{th}\,\theta} - 1) + i[(\frac{\theta}{\mathrm{th}\,\theta} - 1)\ln\frac{4(\mathrm{ch}\,\theta + 1)^2}{\gamma^2\lambda(\mathrm{ch}\,\theta - 1)} + 2 - \ln 2 - \mathrm{ch}\,\theta\, R(\theta)]\}, \tag{24}$$

$$\mathit{\Delta} W_0 = \frac{e^2}{8\pi^2}\{\pi(1 - \frac{\theta}{\mathrm{sh}\,\theta}) + i[(1 - \frac{\theta}{\mathrm{sh}\,\theta})\ln\frac{4(\mathrm{ch}\,\theta + 1)^2}{\gamma^2\lambda(\mathrm{ch}\,\theta - 1)} - 2 + \ln 2 + R(\theta)]\}, \tag{25}$$

here $\gamma = 1.781,\ R(\theta)$ is even function of θ related to the Euler's dilogarithm $L_2(z)$ [14],

$$R(\theta) = \int_0^\infty d\alpha\,\frac{\ln(\mathrm{ch}\,\theta + \mathrm{ch}\,\alpha)}{\mathrm{ch}\,\theta + \mathrm{ch}\,\alpha} = \frac{L_2(1 - e^{-2\theta}) + \theta^2 - \ln 2\cdot\theta}{\mathrm{sh}\,\theta}. \tag{26}$$

For the case $\theta \to \pm\infty,\ \lambda$ arbitrary, considered in [15,16],

$$\mathit{\Delta} W_{1,0} = -|\,\theta|\frac{e^2}{8\pi^2}\,S_{1,0}(\lambda), \quad S_n(\lambda) = (-1)^{n+1}\int_0^\infty dz\, e^{-i\lambda/2z}[e^{iz}\,K_n(iz) - \sqrt{\frac{\pi}{2iz}}], \tag{27}$$

where $K_n(iz)$ is the Mcdonald function. At $\lambda \to 0$

$$S_1(\lambda) = -\pi - i(\ln\frac{4}{\gamma^2\lambda} - 1), \quad S_0(\lambda) = -i. \tag{28}$$

For the trajectory with subluminal relative velocity β_{12} of the ends the $\mathrm{Re}\,\mathit{\Delta} W_{1,0}$ are given by the formulae

$$\mathrm{Re}\,\mathit{\Delta} W_1 = \frac{e^2}{8\pi}(\frac{\theta}{\mathrm{th}\,\theta} - 1), \qquad \mathrm{Re}\,\mathit{\Delta} W_0 = \frac{e^2}{8\pi}(1 - \frac{\theta}{\mathrm{sh}\,\theta}). \tag{29}$$

When $\beta_{12} \to 1$ the trajectory becomes actually hyperbolic one with charge's velocity $\beta(\tau) = -\mathrm{th}\,w_0\tau$ at proper time τ, and $\theta = w_0(\tau_2 - \tau_1) \to \infty$. Then

$$\mathrm{Re}\,\mathit{\Delta} W_1 = \frac{e^2 w_0}{8\pi}\,(\tau_2 - \tau_1), \qquad \mathrm{Re}\,\mathit{\Delta} W_0 = \frac{e^2}{8\pi}, \tag{30}$$

while the mass shifts of uniformly accelerated charges are

$$\mathit{\Delta} m = -\frac{\partial \mathit{\Delta} W}{\partial \tau_2} = \frac{e^2 w_0}{8\pi^2}\,S(\lambda); \quad \text{at } \lambda \to 0 \quad \mathrm{Re}\,\mathit{\Delta} m_1 = -\frac{e^2 w_0}{8\pi}, \quad \mathrm{Re}\,\mathit{\Delta} m_0 = 0. \tag{31}$$

The real parts of the action changes (29) have interesting integral representations ascending to Legendre [17]

$$\mathrm{Re}\,\mathit{\Delta} W_{1,0} = \frac{e^2}{4\pi}\int_0^\infty dx\,\frac{\sin\,x}{e^{\pi x/\theta}\mp 1}, \qquad \theta = \mathrm{Arth}\,\beta_{12}. \tag{32}$$

If β_{12} is close to 1 then on the large interval of the quasihyperbolic trajectory the velocity $\beta(\tau) \approx -\mathrm{th}\,w_0\tau$, i.e. is the same as for hyperbolic trajectory, and the parameter $\theta \approx w_0(\tau_2 - \tau_1)$, where $\mathit{\Delta}\tau = \tau_2 - \tau_1$ is the proper time interval inside of which the charge moves with acceleration w_0 and outside - with constant initial and final velocities $\beta_1,\ \beta_2$.

On the acceleration interval the mass shift of a charge can be defined by one of two relations

$$\mathrm{Re}\,\mathit{\Delta} m = -\frac{\partial\,\mathrm{Re}\,\mathit{\Delta} W}{\partial\,\tau_2}, \qquad \mathrm{Re}\,\widetilde{\mathit{\Delta} m} = -\frac{\mathrm{Re}\,\mathit{\Delta} W}{\mathit{\Delta}\tau}. \tag{33}$$

By using the Legendre representation and formula $\theta = w_0(\tau_2 - \tau_1)$, we obtain, according to the first definition,

$$\mathrm{Re}\,\mathit{\Delta} m_1 = -\frac{e^2 w_0}{8\pi}\,(\mathrm{cth}\,\theta - \frac{\theta}{\mathrm{sh}^2\,\theta}), \qquad \mathrm{Re}\,\mathit{\Delta} m_0 = -\frac{e^2 w_0}{8\pi}\,\frac{\theta\,\mathrm{cth}\,\theta - 1}{\mathrm{sh}\,\theta},$$
$$\mathrm{Re}\,\mathit{\Delta} m_{1,0} = -e^2\,T\int_0^\infty \frac{d\omega}{2\pi}\,\frac{\sin\,2\omega\mathit{\Delta}\tau}{\omega}\,c^{B,F}(\omega/T), \quad c^{B,F}(z) = \frac{z^2 e^z}{(e^z\mp 1)^2}. \tag{34}$$

In the last, spectral representation $T = w_0/2\pi$ is the Davies-Unruh "temperature" [18,19], while $c^{B,F}(\omega/T)$ are the heat capacity spectral densities of Bose- and Fermi-gases of massless particles in one-dimensional space, see Sections 49, 105 in [20].

The $\mathrm{Re}\,\mathit{\Delta} m_{1,0} \leqslant 0$ for all finite $\theta \geqslant 0$; for $\theta \ll 1$

$$\mathrm{Re}\,\mathit{\Delta} m_1 = 2\,\mathrm{Re}\,\mathit{\Delta} m_0 = -\frac{e^2 w_0}{8\pi}\,\frac{2}{3}\,\theta, \tag{35}$$

and for $\theta \to \infty$

$$\mathrm{Re}\,\mathit{\Delta} m_1 = -\frac{e^2 w_0}{8\pi}, \qquad \mathrm{Re}\,\mathit{\Delta} m_0 = 0. \tag{36}$$

Note, that when the duration of acceleration $\mathit{\Delta}\tau \to \infty$, the functon

$$\frac{\sin\,2\omega\mathit{\Delta}\tau}{\omega}|_{\mathit{\Delta}\tau\to\infty} = \pi\,\delta(\omega). \tag{37}$$

The function on the left-hand side is the Fourier-transform of the acceleration switching function. The acceleration interval can be regulated by the scale-changing of the "temperature" parameter and the frequency, $T \to kT,\ \omega \to k\omega$, at constant ratio ω/T. Thus, the temperature $T = 2w_0/\pi$ also can be used [16].

According to the second definition

$$\mathrm{Re}\,\widetilde{\mathit{\Delta} m_1} = -\frac{e^2 w_0}{8\pi}\,(\mathrm{cth}\,\theta - \frac{1}{\theta}), \qquad \mathrm{Re}\,\widetilde{\mathit{\Delta} m_0} = -\frac{e^2 w_0}{8\pi}\,(\frac{1}{\theta} - \frac{1}{\mathrm{sh}\,\theta}),$$
$$\mathrm{Re}\,\widetilde{\mathit{\Delta} m_{1,0}} = -e^2\int_0^\infty \frac{d\omega}{2\pi}\,\frac{\sin\,2\omega\mathit{\Delta}\tau}{\omega}\,\frac{\omega}{e^{\omega/T}\mp 1}. \tag{38}$$

In this case the spectral representation contains the energy spectral density of Bose- or Fermi-gas of massless particles in one-dimensional space. The quantities in both representations are connected by the equation

$$\mathrm{Re}\,\varDelta m_{1,0} = T\frac{\partial}{\partial T}\,\mathrm{Re}\,\widetilde{\varDelta m_{1,0}} \tag{39}$$

which is a consequence of the usual relation

$$c^{B,F}(\omega/T) = \frac{\partial}{\partial T}\left(\frac{\omega}{e^{\omega/T}\mp 1}\right) \tag{40}$$

between heat capacity and energy, see Sections 14, 42 in [20].

As a functions of θ the shifts $\mathrm{Re}\,\varDelta m_{1,0}$ and $\mathrm{Re}\,\widetilde{\varDelta m_{1,0}}$ differ in magnitude at $\theta \lesssim 1$, for example,

$$\mathrm{Re}\,\varDelta m_{1,0} = 2\,\mathrm{Re}\,\widetilde{\varDelta m_{1,0}}, \qquad \theta \ll 1, \tag{41}$$

but have the same limiting values (36) as $\theta \to \infty$. It can be understood, for the mass shift formation it is necessary proper time not less than inverse acceleration w_0^{-1}.

Thus, according to the spectral representations (34), (38) the symmetry being discussed reveals itself also in the formation of the mass shifts of electric and scalar charges at acceleration. The vector and scalar massless Bose-fields of the charges in 3+1-space again appear to be connected with the massless scalar (Bose) and spinor (Fermi) fields in 1+1-space. The symmetry explaines why the Legendre representations of the self-action changes and mass shifts of electric and scalar charges, the sources of the Bose-fields, contain the spectral distributions characteristic for Bose- and Fermi-fields in one-dimensional space.

The symmetry explaines the limiting values (36) of the mass shifts $\mathrm{Re}\,\varDelta m_{1,0}$ for uniformly accelerated electric and scalar charges in 3+1-space by the nonzero and zero low-frequency limits of the heat capacity (or energy) spectral densities for Bose and Fermi gases in 1+1-space:

$$c^{B,F}(\omega/T)|_{\omega=0} = 1,\ 0; \qquad u^{B,F}(\omega) = \frac{\omega}{e^{\omega/T}\mp 1}|_{\omega=0} = T,\ 0. \tag{42}$$

The appearance of the heat quantum-mechanical distributions in the spectral representations of the dynamical mass shifts $\varDelta m_{1,0}$ is no less intriguing than their appearance in the Hawking effect [1], especially when the absence of the horizons for the quasihyperbolic trajectory is taken into account.

According to spectral formulae for $\varDelta m$ and $\widetilde{\varDelta m}$ the proper field energy of the charges decreases at the acceleration due to radiation on the frequencies

$$\omega_n = \frac{\pi(n+1/2)}{2\,\varDelta\tau} \tag{43}$$

with even $n = 0,\, 2,\, \ldots,$ and increases due to excitation on the frequencies ω_n with odd $n = 1,\, 3,\, \ldots.$ We can say that the proper field releases (deconfines) the excitations with even n and confines the ones with odd n. The rescaling of T and ω does not change this assertion. Eventually, for every finite $\theta > 0$ the radiation-excitation balance produces the $\mathrm{Re}\,\varDelta m < 0$, and the $\mathrm{Re}\,\varDelta W > 0,\ \mathrm{Im}\,\varDelta W > 0$.

Simultaneous radiation and excitation of the proper field of a charges at acceleration is supported by the positive and negative contributions with even and odd n frequencies ω_n to the imaginary part of the self-action change,

$$\mathrm{Im}\,\Delta W = \frac{1}{\pi}\mathrm{Re}\,\Delta W \cdot \ln\frac{1}{\lambda} + \cdots\,, \tag{44}$$

more precisely, to its principal, infrared part, see (24),(25).

Due to the symmetry the quantities $\Delta W^{B,F}$, $\Delta m^{B,F}$ for the mirror interacting with massless Bose- or Fermi-field can be obtained from $\Delta W_{1,0}$, $\Delta m_{1,0}$ by the change $e^2 \to \hbar c$.

4 Arguments in favour of the value $\alpha_0 = 1/4\pi$ for the bare fine structure constant

At the collisions of a charged particles, for example, two electrons, the emission of soft photons takes place, which does not affect the motion of the colliding charges. As a result, the cross-section of the particle scattering with the emission of n soft photons is represented by the formula (98.21) of [23]:

$$d\sigma = d\sigma_{scat}\, w(n), \quad w(n) = \frac{\bar{n}^n}{n!}\, e^{-\bar{n}}, \tag{45}$$

where $w(n)$ is the probability of emission of n soft photons in appropriate frequency interval (ω_1, ω_2) and $\bar{n}$ is their mean number, which can be found by classical electrodynamics. In this paper the vacuum-vacuum amplitude is considered which modulus squared is equal to $w(0) = e^{-\bar{n}}$.

It is important that the main, logarithmic term of $\bar{n}$,

$$\bar{n} = \alpha\,\frac{2}{\pi}\,(\theta\,\mathrm{cth}\,\theta - 1)(\ln\frac{\omega_2}{\omega_1} + f(\theta)), \tag{46}$$

(see Sections 98, 120 in [23], and formula (24) in this paper where $2\,\mathrm{Im}\,\Delta W_1 = \bar{n}$, $\mu = \omega_1$, $w_0 = \omega_2$), does not depend on the details of the charge motion and is defined by the invariant momentum transfer $\xi = q/2m = \sqrt{t}/2m = \mathrm{sh}\,(\theta/2)$, which together with the total energy $\sqrt{-s}$ defines the main hard process. Thus, independently of the charge's motion ("trajectory") inside the forming region of the hard process, the mean number of photons emitted by the charge is defined by the global parameter – the momentum transfer or the Lorentz invariant velocity change β_{12} of a charge in the mentioned region, $\theta = \mathrm{Arth}\,\beta_{12}$.

The $w(0) = e^{-\bar{n}}$ with the main, logarithmic term for $\bar{n}$ is given by Abrikosov formula (136.11) in [23] for high energy and momentum transfer . It coincides with (46) where $\omega_1 = \omega_m$, $\omega_2 = \varepsilon$. In Abrikosov approximation the effective (running) fine structure constant $\alpha_{eff}(q^2)$ [22,23] does not differ from α,

$$\alpha_{eff}(q^2) = \frac{\alpha}{1 - (\alpha/3\pi)\,N_i\,\ln{(q^2/m_i^2)}}. \tag{47}$$

Here m_i and N_i are the masses and numbers of different type vacuum charges screening the bare charge, $m_i < q$. For super high momentum transfers this formula does not work.

If the variant (b) of the Gell-Mann and Low paper [21] is realized in quantum electrodynamics, then on the distances less than some supersmall $\Lambda^{-1} \ll m^{-1}$ the QED is characterized by the finite point bare charge e_0 and the charge density $e_0\,\delta\,(\mathbf{x})$. In more detail, if the bare fine structure constant $\alpha_0 = e_0^2/4\pi\hbar c$ is finite, then [21]

1) it does not depend on the value of the fine structure constant α,

2) the α must be less than α_0, and

3) the charge density at very small distances reduces to the delta-function $e_0\delta(\mathbf{x})$.

Therefore, at a collision of a charges with total energy $\sqrt{-s} = 2E$ and momentum transfer $\sqrt{t} \approx 2E \gg \Lambda$ the cross-section $d\sigma_{scat}$ will be defined by the bare charge e_0, and $\bar{n}$ will be given by the formula

$$\bar{n} = \alpha_0\,\frac{2}{\pi}\,(\theta\,\mathrm{cth}\,\theta - 1)(\ln\frac{\omega_2}{\omega_1} + f(\theta)), \tag{48}$$

if the frequencies ω_1, ω_2 satisfy the condition $\Lambda \lesssim \omega_1 \ll \omega_2 \ll E$. In this case the photon emission does not influence as before on the dynamics of the hard process though comes from the super small region $\sim \Lambda^{-1}$ where the charge is pointlike and equals e_0. Under these conditions the motion of each colliding charge is one-dimensional and can be approximate by classical trajectory with fixed $\theta = \mathrm{Arth}\,\beta_{12}$ related to this super small region.

The symmetry discussed consists in the coincidence of the number spectrum of pairs of Bose (Fermi) massless quanta, emitted by point mirror in 1+1-space, with the number spectrum of photons (massless scalar quanta), emitted by point electric (scalar) charge. The first one is obtained by quantum field theory with the corresponding zero boundary condition on the mirror, while the second is obtained by the division of classical energy spectrum on $\hbar\omega$. The corresponding spectra coincide as a functions of two variables and a functionals of the any common trajectory of the mirror and the charge. The only distinction in multiplier $e^2/\hbar c$ can be removed if one puts $e^2 = \hbar c$.

This symmetry is a consequence of

1) the invariant structure of scalar products in quantum theory of scalar and spinor fields,

2) the pointness of the mirror and the charge,

3) the unaffectness of the quantum emission on the mirror and charge motions,

4) the space one-dimensionality of the motion.

The 2-dimensional model of QFT with a point mirror interacting with the secondary quantized Bose (Fermi)-massless field [4] is pure geometrical: it has no mass-dimensional parameters and its Planck constant is dimensionless and equals 1. The usual Planck constant appears in the comparison of this QFT-model results with the results of QED where there are charge, mass, and momenta and energies instead of wave vectors and frequencies, or with the results of classical electrodynamics where there are charge, mass and the energy of radiation.

The dimensionless multiplier $e^2/\hbar c$, by which the number spectrum of soft photons ($\hbar\omega \ll mc^2$ in proper system of a charge) in QED differs from the number spectrum of Bose-pairs in 2-dimensional QFT-model, is less than 1 because the charge in QED has finite dimensions $\sim \hbar/mc$ due to the screening, while the mirror, the source of Bose-pairs, is point-like.

If for the superhigh energy and momentum transfer QED has finite charge e_0 of vanishingly small dimensions, then these dimensions cannot be defined better than inverse energy $\hbar c/\sqrt{-s}$ of two head-on colliding charges. Therefore, it is reasonable to assume that at $\sqrt{-s} \approx \sqrt{t} \to \infty$ the spectrum of photons with frequencies $\Lambda \lesssim \hbar\omega \ll \sqrt{-s}$ emitted by

the bare charge e_0 does not differ from the spectrum of Bose-pairs radiated by point mirror. Then $e_0^2 = \hbar c$ and $\alpha_0 = 1/4\pi$. The Gell-Mann and Low properties of α_0 are fulfilled.

Let us consider the head-on collision of two electrons with mass m, charge e, and very large energy E on infinity. The elastic scattering cross-section depends on two invariants, s and t, which in the center of mass system are equal to

$$s = -4E^2, \quad t = 2p^2(1 - \cos\vartheta),$$

where $E = \sqrt{p^2 + m^2}$, p and ϑ are the energy, momentum and scattering angle of electron in c.m.s. At fixed energy E the smallest distance between the charges is attained at the largest momentum transfer, i.e. at $\vartheta = \pi$, when charges move along the same straight line. Namely in this case each of them most deeply penetrates under the screening coat of the other. Suppose that the total energy is enough to penetrate into the region where the electron charges become bare. Then the minimal distance between them is equal to

$$r^c_{min} = \frac{\alpha_0}{2E} \tag{49}$$

according to classical theory.

But according to quantum mechanics at the distance r between the charges the uncertainty in their momentum will be not less than $\Delta p \approx 1/r$. It may be thought that the charges can not be on the distance less than r^q_{min} at which the momentum uncertainty would lead to the energy greater than $2E$. Then $2E = 2\sqrt{M^2 + \Delta p^2}$, $\Delta p = \sqrt{E^2 - M^2}$, where M is the mass of the bare charge. The Gell-Mann and Low's pointness of the bare charge forces one to consider that $M \sim E$.

For the r^q_{min} we have

$$r^q_{min} = \frac{1}{\Delta p} = \frac{1}{\sqrt{E^2 - M^2}} = \frac{2E}{\alpha_0\sqrt{E^2 - M^2}}\, r^c_{min}. \tag{50}$$

As the minimal quantum distance is distinctly larger than the classical one, the turning point can be considered to be defined namely by the r^q_{min}. Then the proper acceleration of the charge at the turning point can be found from the equation

$$M\, w_0 = \frac{e_0^2}{4\pi\, r^{q\,2}_{min}} = \alpha_0\,(E^2 - M^2). \tag{51}$$

The quantum motion of the charges is little affected by the emission of photons with frequencies not greater than w_0 as the ratio

$$\frac{w_0}{E} = \alpha_0\,\frac{E^2 - M^2}{EM} \tag{52}$$

is small if the α_0 is small and $M \sim E$. Therefore, for the calculation of soft photon emission such motion can be approximated by the classical trajectory with acceleration w_0 at the turning point. As a result, for the $\bar{n}$ one obtains the formula (48), where $\omega_2 = w_0$, and the parameter θ

$$\theta = 2\,\mathrm{Arsh}\,\frac{q}{2M}, \quad q = \Delta p = \sqrt{E^2 - M^2}. \tag{53}$$

At $M \sim E$ the force acting on the charge e_0 according to (51) is of the order of $\alpha_0 M^2$ and is small in comparison both with classical force M^2/α_0, when classical electrodynamics becomes self-contradictory, and with quantum force M^2, when QED needs in quantum corrections, see Sec. 75 in [24].

About 45 years ago E.P. Wigner remarked that the special relativity is the physics of Lorentz transformation, and the quantum mechanics is the physics of Fourier transformation. Processes induced by a point mirror in 1+1-space are described by the simplest relativistic quantum theory, which is incarnated in Bogoliubov coefficients. They are Lorentz-invariant scalar products reduced to Fourier transforms of massless scalar and spinor wave equation solutions. They can be considered as concentrate of genetic information about processes in 3+1- space.

5 Self-action changes $\varDelta W_{1,0}$ and traces $\mathrm{tr}\,\alpha^{B,F}$

The basis for the symmetry between the processes induced by the mirror in two-dimensional and by the charge in four-dimensional space-time is the relation (11), (12) between the Bogoliubov's coefficients $\beta^{B,F}_{\omega'\omega}$ and the current density $j^\alpha(k)$ or charge density $\rho(k)$ depending on the timelike momentum k^α. The squares of these quantities represent the spectra of real pairs and particles radiated by accelerated mirror and charge.

The symmetry is extended to the selfactions of the mirror and the charge and to the corresponding vacuum-vacuum amplitudes, cf. (18) and (19). In essence, it is embodied in the integral relation (16) between propagators of a massive pair in two-dimensional space and of a single particle in four-dimensional space.

The formula (18) for $W^{B,F}$ was obtained provided that the mean number $\bar{N}^{B,F}$ of pairs created is small and the interference of two or more pairs is negligible. In the general case the $W^{B,F}$ is given by the formula (13), which can be written also in the form

$$2\,\mathrm{Im}\,W^{B,F} = \pm \mathrm{tr}\,\ln(\alpha^+\alpha)^{B,F}, \tag{54}$$

since $\alpha^+\alpha \mp \beta^+\beta = 1$, see [4], [6]. As is seen from (13), the imaginary part of the action differs from zero and then is positive only if $\beta \neq 0$, i.e. if the radiation of real particles is happened indeed.

Formula (54) allows to choose for $W^{B,F}$ the expression

$$W^{B,F} = \pm i\,\mathrm{tr}\,\ln\,\alpha^{B,F}, \tag{55}$$

that was called natural by DeWitt [4]. However, this expression is by no means unique, the expressions with $\alpha e^{i\gamma}$ or α^+ have the same imaginary part. Nevertheless, the formula (55) is interesting as the definition both the real and imaginary parts of the selfactions $W^{B,F}$ by means of the Bogoliubov's coefficients $\alpha^{B,F}_{\omega'\omega}$ only, which, according to the formulae (11), (12), reduce to the current density $j^\alpha(q)$ or to the charge density $\rho(q)$ dependent on the spacelike momentum q^α. This means that the field of the corresponding perturbations propagates in vacuum together with the mirror, comoves it, and, at the same time, it contains the information about the radiation of the real quanta.

Unfortunately, the author failed to find a simple integral representation for the matrix $\ln\,\alpha$. Nevertheless, if one again assumes that the mean number of emitted particles is small,

then one may consider α, or $i\alpha$, or $\pm i\alpha^{B,F}$ close to 1. Namely the last phase factor is most acceptable as will be seen below. Then, expanding the $\ln(\pm i\alpha^{B,F})$ near $\pm i\alpha^{B,F}=1$ and confine ourselves by the first term we obtain

$$W^{B,F}=\pm i\,\mathrm{tr}\,\ln(\pm i\alpha^{B,F})\sim\pm i\,\mathrm{tr}\,(\pm i\alpha^{B,F}-1)=-\mathrm{tr}\,\alpha^{B,F}+\dots\,. \tag{56}$$

These qualitative arguments allow to state that the functionals $\mathrm{tr}\,\alpha^{B,F}$ are similar to the corresponding self-actions with opposite sign and therefore must have the negative imaginary parts. This is confirmed by the general examples considered below in which at least the initial or the final velocity of the mirror is subluminal.

However, as is shown in the next Section, the above reasoning is very crude. The exact physical meaning of the $\mathrm{tr}\,\alpha^{B,F}$ is conveyed by the formula (99) or (102). As a result, each of the traces represents the mass shift of a field, entrained by a mirror at acceleration, multiplied by effective proper time of the shift formation. This time is of the order of w_0^{-1}.

6 Invariant structure of Bogoliubov coefficients

Here, using, as an example, the Bogoliubov coefficients for hyperbolic motion of the mirror [25,26],

$$\alpha^{B,F}_{\omega'\omega}=\frac{2}{\sqrt{\varkappa\varkappa'}}\,e^{i(\frac{\omega}{\varkappa}+\frac{\omega'}{\varkappa'})}K_{1,0}(2i\sqrt{\frac{\omega\omega'}{\varkappa\varkappa'}})=\frac{2}{w_0}\,e^{i\frac{\rho}{w_0}\mathrm{ch}\,(\vartheta-\alpha)}K_{1,0}(\frac{i\rho}{w_0}). \tag{57}$$

$$\beta^{B,F*}_{\omega'\omega}=(-i)^{1,0}\frac{2}{\sqrt{\varkappa\varkappa'}}\,e^{i(-\frac{\omega}{\varkappa}+\frac{\omega'}{\varkappa'})}K_{1,0}(2\sqrt{\frac{\omega\omega'}{\varkappa\varkappa'}})=(-i)^{1,0}\frac{2}{w_0}\,e^{-i\frac{\rho}{w_0}\mathrm{sh}\,(\vartheta-\alpha)}K_{1,0}(\frac{\rho}{w_0}). \tag{58}$$

we shall consider the invariant properties of the coefficients relative to Lorentz transformation and the transformation properties relative to transfer of the reference origin from one point on the trajectory to another.

The Bogoliubov coefficients are the functionals of the trajectory and a functions of the frequencies ω, ω' and parameters $\varkappa$, $\varkappa'$. The latters characterize the mirror trajectory $u^{mir}=g(v)$ near the coordinate origin $u=v=0$ chosen on the trajectory:

$$u^{mir}=g(v)=\frac{1}{\varkappa}\left(\varkappa'v+b(\varkappa'v)^2+\frac{1}{3}c(\varkappa'v)^3+\cdots\right). \tag{59}$$

The velocity and proper acceleration of the mirror at the point $u=v=0$ are equal to

$$\beta_0=\frac{1-\varkappa'/\varkappa}{1+\varkappa'/\varkappa},\qquad a_0=-b\sqrt{\varkappa\varkappa'}. \tag{60}$$

At the Lorentz transformation with velocity $\beta=\mathrm{th}\,\delta$ parameters $\varkappa$, $\varkappa'$ are transformed just as frequencies ω, ω':

$$\tilde{\omega}=\frac{\omega-\beta\,\omega}{\sqrt{1-\beta^2}}=\omega\,e^{-\delta},\quad \tilde{\omega}'=\frac{\omega'+\beta\,\omega'}{\sqrt{1-\beta^2}}=\omega'\,e^{\delta}, \tag{61}$$

and the product $\tilde{\omega}\tilde{\omega}' = \omega\omega'$ is invariant. Therefore the frequencies ω, ω' and parameters $\varkappa$, $\varkappa'$ can be represented in the form

$$\omega = \sqrt{\omega\omega'}\, e^{\vartheta},\ \omega' = \sqrt{\omega\omega'}\, e^{-\vartheta}; \quad \varkappa = \sqrt{\varkappa\varkappa'}\, e^{\alpha},\ \varkappa' = \sqrt{\varkappa\varkappa'}\, e^{-\alpha}. \tag{62}$$

In the coordinate system with velocity β_C relative to the laboratory system

$$\beta = \beta_C = \frac{\omega - \omega'}{\omega + \omega'} = \mathrm{th}\,\vartheta, \qquad \vartheta = \ln\sqrt{\omega/\omega'}, \tag{63}$$

the frequencies ω and ω' of reflected and incident waves coincide and are equal to invariant $\sqrt{\omega\omega'}$ while the vectors

$$k^{\alpha} = (k^1,\, k^0) = (\omega - \omega',\, \omega + \omega'), \quad q^{\alpha} = (q^1,\, q^0) = (-\omega - \omega',\, -\omega + \omega'), \tag{64}$$

have only time and only space components correspondingly:

$$k^{\alpha}_C = (0,\, 2\sqrt{\omega\omega'}), \qquad q^{\alpha}_C = (-2\sqrt{\omega\omega'},\, 0). \tag{65}$$

The given formulae were used in the coefficients (43), (44) for hyperbolic trajectory

$$t(\tau) = \frac{\mathrm{sh}\,(w_0\tau - \alpha) + \mathrm{sh}\,\alpha}{w_0}, \qquad x(\tau) = \frac{\mathrm{ch}\,\alpha - \mathrm{ch}\,(w_0\tau - \alpha)}{w_0}, \tag{66}$$

for which the proper acceleration is equal to $a_0 = -\sqrt{\varkappa\varkappa'} = -w_0$.

The velocity of the mirror on this trajectory at the moment $w_0\tau$ is equal to

$$\beta(w_0\tau) = \frac{\dot{x}(w_0\tau)}{\dot{t}(w_0\tau)} = -\mathrm{th}\,(w_0\tau - \alpha). \tag{67}$$

The mirror passes the coordinate origin with velocity $\beta_0 = \beta(0) = \mathrm{th}\,\alpha$ at the moment $w_0\tau = 0$, it passes the turning point at the moment $w_0\tau = \alpha,\ \beta(\alpha) = 0$, and at the moments $w_0\tau_{1,2} = \alpha \mp (\vartheta - \alpha)$ before and after the turn its velocities are equal to

$$\beta(w_0\tau_{1,2}) = \pm\mathrm{th}\,(\vartheta - \alpha) = \pm\frac{\beta_C - \beta_0}{1 - \beta_C\beta_0} = \pm\beta_{C0}. \tag{68}$$

The velocities β_C and β_{C0} are the velocities of the pair of waves ω, ω' in the laboratory system and in the system moving relative to the laboratory system with velocity β_0. This last system will be called the system of detector which moves with constant velocity β_0 and touches to the mirror at the point $t = x = 0$.

Thus, the laboratory time intervals

$$\Delta t_{1,2} = t(w_0\tau_{1,2}) - t(\alpha) = \mp\frac{\mathrm{sh}\,(\vartheta - \alpha)}{w_0} \tag{69}$$

and the laboratory space intervals

$$\Delta x_{1,2} = x(w_0\tau_{1,2}) - x(\alpha) = -\frac{\mathrm{ch}\,(\vartheta - \alpha) - 1}{w_0} \tag{70}$$

marked off from the turning point define the time and length of deceleration ($w_0\tau_1 \leqslant w_0\tau \leqslant \alpha$) and acceleration ($\alpha \leqslant w_0\tau \leqslant w_0\tau_2$) intervals on the world trajectory of the mirror where its velocity changes monotonously in the interval

$$-\beta_{C0} \leqslant \beta \leqslant \beta_{C0} \tag{71}$$

between opposite in sign values (60) and takes zero value at the turning point. It is supposed that $\vartheta > \alpha$. In the case $\vartheta < \alpha$ the moment $\tau_2 < \tau_1$ and the deceleration and acceleration intervals will be $w_0\tau_2 \leqslant w_0\tau \leqslant \alpha$ and $\alpha \leqslant w_0\tau \leqslant w_0\tau_1$ correspondingly.

Let us show that the intervals $\mathit{\Delta} t_{1,2}$ and $\mathit{\Delta} x_{1,2}$ are Lorentz-invariants, i.e. do not depend on transition to another Lorentz coordinate system. Let the system $\tilde{K}$ moves with velocity $\beta = \operatorname{th}\delta$ relative to the laboratory system K. Then the mirror motion equations in the system $\tilde{K}$ accept the form

$$\tilde{t}(w_0\tau) = \frac{t(w_0\tau) - \beta\, x(w_0\tau)}{\sqrt{1-\beta^2}} = \frac{\operatorname{sh}(\alpha-\delta) + \operatorname{sh}(w_0\tau - \alpha + \delta)}{w_0}, \tag{72}$$

$$\tilde{x}(w_0\tau) = \frac{x(w_0\tau) - \beta\, t(w_0\tau)}{\sqrt{1-\beta^2}} = \frac{\operatorname{ch}(\alpha-\delta) + \operatorname{ch}(w_0\tau - \alpha + \delta)}{w_0}, \tag{73}$$

differing from the nontransformed ones by the shift $\alpha \to \tilde{\alpha} = \alpha - \delta$ of the parameter α.

The velocity of the mirror in new system is

$$\tilde{\beta}(w_0\tau) = \frac{\dot{\tilde{x}}(w_0\tau)}{\dot{\tilde{t}}(w_0\tau)} = -\operatorname{th}(w_0\tau - \alpha + \delta). \tag{74}$$

At the moment $w_0\tau = 0$ of passage through the origin the velocity is equal to $\tilde{\beta}_0 = \tilde{\beta}(0) = \operatorname{th}(\alpha-\delta)$; the turning point is went through at the moment $w_0\tau = \alpha - \delta$.

Since the frequencies ω, ω' at the Lorentz-transformation with velocity $\beta = \operatorname{th}\delta$ go over into the frequencies $\tilde{\omega}$, $\tilde{\omega}'$,

$$\tilde{\omega} = \sqrt{\omega\omega'}\, e^{\vartheta-\delta}, \quad \tilde{\omega}' = \sqrt{\omega\omega'}\, e^{-\vartheta+\delta}, \tag{75}$$

and differ from nontransformed ones by the shift $\vartheta \to \tilde{\vartheta} = \vartheta - \delta$ of the parameter ϑ, the velocity $\beta_C = \operatorname{th}\vartheta$ of the pair of waves ω, ω' goes into the velocity $\tilde{\beta}_C = \operatorname{th}(\vartheta-\delta)$ of the Lorentz-transformed pair of waves $\tilde{\omega}$, $\tilde{\omega}'$. However, the relative velocity of this pair of waves and detector,

$$\tilde{\beta}_{C0} = \frac{\tilde{\beta}_C - \tilde{\beta}_0}{1 - \tilde{\beta}_C\tilde{\beta}_0} = \operatorname{th}(\vartheta-\alpha) = \beta_{C0}, \tag{76}$$

remains unchanges because $\tilde{\vartheta} - \tilde{\alpha} = \vartheta - \alpha$, see (62).

In the new system the time and length of the intervals of deceleration ($w_0\tilde{\tau}_1 = 2\alpha - \vartheta - \delta \leqslant w_0\tau \leqslant \alpha - \delta$) and acceleration ($\alpha - \delta \leqslant w_0\tau \leqslant w_0\tilde{\tau}_2 = \vartheta - \delta$) from the same initial velocity $\tilde{\beta}(w_0\tilde{\tau}_1) = \operatorname{th}(\vartheta-\alpha)$ to the same final velocity $\tilde{\beta}(w_0\tilde{\tau}_2) = -\operatorname{th}(\vartheta-\alpha)$ do not depend on parameter δ and remain the former functions of the Lorentz-invariant difference $\vartheta - \alpha = \tilde{\vartheta} - \tilde{\alpha}$:

$$\mathit{\Delta}\tilde{t}_{1,2} = \tilde{t}(w_0\tilde{\tau}_{1,2}) - \tilde{t}(\alpha-\delta) = \mp\frac{\operatorname{sh}(\vartheta-\alpha)}{w_0} = \mathit{\Delta} t_{1,2}, \tag{77}$$

$$\Delta\tilde{x}_{1,2} = \tilde{x}(w_0\tilde{\tau}_{1,2}) - \tilde{x}(\alpha-\delta) = -\frac{\mathrm{ch}\,(\vartheta-\alpha)-1}{w_0} = \Delta x_{1,2}. \quad (78)$$

This difference is nothing but the proper time (multiplied by w_0) of the mentioned deceleration or acceleration.

For the parameter $\delta = \alpha$ the system $\tilde{K}$ moves with velocity β_0 relative to the laboratory system and coincides with the proper system of the detector which the mirror touches to at its turning point $\tilde{t} = \tilde{x} = 0$. The frequencies $\tilde{\omega}$, $\tilde{\omega}'$ of the waves of pair in the detector system will be denoted as Ω, Ω':

$$\Omega = \omega\sqrt{\frac{\varkappa'}{\varkappa}}, \qquad \Omega' = \omega'\sqrt{\frac{\varkappa}{\varkappa'}}. \quad (79)$$

Evidently, they are Lorentz-invariant quantities.

In this system $\tilde{\beta}_0 = 0$, and the invariant relative velocity

$$\beta_{C0} = \tilde{\beta}_{C0} = \tilde{\beta}_C = \frac{\Omega-\Omega'}{\Omega+\Omega'} = \mathrm{th}\,\Theta = \mathrm{th}\,(\vartheta-\alpha), \quad \Theta = \ln\sqrt{\frac{\Omega'}{\Omega}} = \ln\sqrt{\frac{\omega\varkappa'}{\omega'\varkappa}} = \vartheta-\alpha, \quad (80)$$

coincides with the velocity $\tilde{\beta}_C$ of pair of waves Ω, Ω' and is defined by the ratio Ω/Ω' of the transformed frequencies only.

The intervals $\Delta\tilde{t}_{1,2}$, $\Delta\tilde{x}_{1,2}$ are given by the formulae (77,78), where $\delta = \alpha$, $w_0\tilde{\tau}_{1,2} = \mp(\vartheta-\alpha)$ and $\tilde{t}(0) = \tilde{x}(0) = 0$. Therefore,

$$\Delta\tilde{t}_{1,2} = \tilde{t}(w_0\tilde{\tau}_{1,2}) = \mp\frac{\mathrm{sh}\,(\vartheta-\alpha)}{w_0} = \Delta t_{1,2}, \quad (81)$$

$$\Delta\tilde{x}_{1,2} = \tilde{x}(w_0\tilde{\tau}_{1,2}) = -\frac{\mathrm{ch}\,(\vartheta-\alpha)-1}{w_0} = \Delta x_{1,2}. \quad (82)$$

At the switching off the acceleration the trajectory of the mirror coincides with the trajectory of the detector and the $\alpha_{\omega'\omega}$ becomes the matrix diagonal in frequencies (79):

$$\alpha^{B,F}_{\omega'\omega} = 2\pi\,\delta(\Omega-\Omega'). \quad (83)$$

Its functional dependence on the trajectory reduces in this case to the dependence on the parameter $\beta_0 = \mathrm{th}\,\alpha = \mathrm{th}\,(\ln\sqrt{\varkappa/\varkappa'})$ or the Doppler factor $\sqrt{\varkappa/\varkappa'}$ entering into Ω, Ω'.

In the absence of acceleration the frequencies ω, ω' satisfy the condition $\Omega = \Omega'$, and the velocities β_C and β_0 coincide. Acceleration leads to nonzero Bogoliubov coefficients $\beta_{\omega'\omega} \neq 0$ and to the absence of the connection $\Omega = \Omega'$ or $\beta_C = \beta_0$. Distinction between the frequencies Ω, Ω' or velocities β_C, β_0 can be described by the invariant relative velocity β_{C0}, see (68) and (76), and leads to the appearance of invariant phases of Bogoliubov coefficients defined by this parameter.

With the help of intervals (69,70) the Bogoliubov coefficients can be written in the form

$$\alpha^{B,F}_{\omega'\omega} = \frac{2}{w_0}\,e^{-i\rho\Delta x_2+i\rho/w_0}K_{1,0}(\frac{i\rho}{w_0}), \qquad \beta^{B,F}_{\omega'\omega} = \frac{2(-i)^{1,0}}{w_0}\,e^{-i\rho\Delta t_2}K_{1,0}(\frac{\rho}{w_0}), \quad (84)$$

i.e. in the form of proper functions of invariant operators $-i\partial/\partial\Delta x_2$ and $i\partial/\partial\Delta t_2$:

$$-i\frac{\partial\alpha}{\partial\Delta x_2} = -\rho\,\alpha, \qquad i\frac{\partial\beta^*}{\partial\Delta t_2} = \rho\,\beta^*, \tag{85}$$

with invariant proper values of momentum transfer $-\rho$ and mass ρ correspondingly.

Thus the phases of the coefficients (84) are defined by the length $\Delta x_{1,2}$ or the time $\Delta t_{1,2}$ of mirror motion near the turning point where the velocity of the mirror changes its sign and does not exceed in magnitude the velocity of pair created with time-like momentum.

In one and the same laboratory system it can be introduced two coordinate systems K and K', connected by the parallel shift of space-time coordinates

$$x = x_1 + x', \qquad t = t_1 + t'. \tag{86}$$

Monochromatic in- and out-waves in the K and K' systems are differed only by phase multipliers:

$$e^{-i\omega' v} = e^{-i\omega' v_1}\cdot e^{-i\omega' v'}, \qquad e^{-i\omega u} = e^{-i\omega u_1}\cdot e^{-i\omega u'}. \tag{87}$$

Therefore, the Bogoliubov coefficients in the systems K and K' are also differed by phase multipliers:

$$\begin{aligned} \alpha_{\omega'\omega} &= e^{-i(q\Delta)}\cdot\alpha'_{\omega'\omega}, \quad -(q\Delta) = \omega' v_1 - \omega u_1, \\ \beta^*_{\omega'\omega} &= e^{-i(k\Delta)}\cdot\beta'^*_{\omega'\omega}, \quad -(k\Delta) = \omega' v_1 + \omega u_1, \end{aligned} \tag{88}$$

Here $\Delta^\alpha = (x_1, t_1)$ is 2-vector of the shift, and k^α and q^α are the wave 2-vectors (9,10).

Particularly, the origin $x = t = 0$ of coordinate system K can be chosen at the point of the trajectory, where the mirror has nonzero velocity β_0, and the origin $x' = t' = 0$ of the coordinate system K' - at the turning point, where $\beta_1 = 0$. Then x_1, t_1 are the coordinates of the turning point in the K-system. In this case for the hyperbolic trajectory we have

$$u_1 = \frac{1}{w_0} - \frac{1}{\varkappa}, \quad v_1 = \frac{1}{\varkappa'} - \frac{1}{w_0}; \qquad w_0 = \sqrt{\varkappa\varkappa'},\ \beta_0 = \mathrm{th}\,\alpha,\ \alpha = \ln\sqrt{\frac{\varkappa}{\varkappa'}}. \tag{89}$$

And the phases of the corresponding multipliers in (88) are equal to the differences of phases of Bogoliubov coefficients (57,58) with nonzero and zero values of the parameter α:

$$\begin{aligned} -(q\Delta) &= \frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} - \frac{\omega+\omega'}{w_0} = \frac{\rho}{w_0}\mathrm{ch}\,(\vartheta-\alpha) - \frac{\rho}{w_0}\mathrm{ch}\,\vartheta, \\ -(k\Delta) &= -\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} - \frac{-\omega+\omega'}{w_0} = -\frac{\rho}{w_0}\mathrm{sh}\,(\vartheta-\alpha) + \frac{\rho}{w_0}\mathrm{sh}\,\vartheta. \end{aligned} \tag{90}$$

The phases of Bogoliubov coefficients can be written as scalar products

$$\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} = -(q\Delta x), \qquad -\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} = -(k\Delta x), \tag{91}$$

of 2-vectors q^α, k^α defined only by the frequencies ω, ω' and spacelike 2-vector Δx^α defined only by the parameters $\varkappa$, $\varkappa'$:

$$\Delta x^1 = \frac{1}{2\varkappa} + \frac{1}{2\varkappa'}, \quad \Delta x^0 = \frac{1}{2\varkappa'} - \frac{1}{2\varkappa}. \tag{92}$$

The length of $\varDelta x^\alpha$ is equal to $1/\sqrt{\varkappa\varkappa'}$ that is equal to $1/w_0$ for hyperbolic trajectory.

Consequently, we have the following forms for the phases

$$\begin{aligned} -\rho(\varDelta x_2-\frac{1}{w_0}) &= -(q\varDelta x)=\frac{\rho}{w_0}\mathrm{ch}\,(\vartheta-\alpha),\\ -\rho\,\varDelta t_2 &= -(k\varDelta x)=-\frac{\rho}{w_0}\mathrm{sh}\,(\vartheta-\alpha). \end{aligned} \tag{93}$$

Vector $\varDelta x^\alpha$ is closely related to acceleration 2-vector a^α which for the trajectory $u^{mir}=g(v)$ is given by the expression

$$a^\alpha=(a^1,\,a^0)=-\frac{g''}{4g'^2}(1+g',\,1-g'),\qquad g=g(v). \tag{94}$$

At the point $u=v=0$ we get

$$a^\alpha_0=a_0\frac{\varDelta x^\alpha}{\sqrt{\varDelta x^2}},\qquad a_0=-b\sqrt{\varkappa\varkappa'}, \tag{95}$$

where a_0 is the proper acceleration at zero point.

The Lorentz-invariant $\mathrm{tr}\,\alpha$ was defined [26] by the formula

$$\mathrm{tr}\,\alpha=\iint\limits_0^\infty\frac{d\omega d\omega'}{(2\pi)^2}\,\alpha_{\omega'\omega}\,2\pi\,\delta\left(\sqrt{\frac{\varkappa'}{\varkappa}}\omega-\sqrt{\frac{\varkappa}{\varkappa'}}\omega'\right),\qquad \Omega=\sqrt{\frac{\varkappa'}{\varkappa}}\omega,\qquad \Omega'=\sqrt{\frac{\varkappa}{\varkappa'}}\omega', \tag{96}$$

in which the Lorentz-invariant argument of δ-function is the difference of the frequencies Ω and Ω' of reflected and incident waves in the proper system of the mirror at zero point $u=v=0$ where the mirror has velocity β_0 and acceleration $a_0=-b\sqrt{\varkappa\varkappa'}$. The multipliers $\sqrt{\varkappa'/\varkappa}$, $\sqrt{\varkappa/\varkappa'}$ are the Doppler factors connecting the frequencies in the laboratory system and zero point proper system of the mirror (or proper system of the detector).

Thus in the trace formation of matrix α its elements diagonal in invariant frequencies are involved, i.e. the elements $\alpha_{\omega'\omega}$ where $\omega/\varkappa=\omega'/\varkappa'$. Note that matrix elements $\alpha_{\omega'\omega}$, $\beta^*_{\omega'\omega}$ being the scalar functions of the frequencies ω, ω' can be written in the detector system if one performs the changes

$$\begin{aligned} &\omega,\,\omega'\to\Omega,\,\Omega';\ u,\,v\to U=\sqrt{\frac{\varkappa}{\varkappa'}}u,\,V=\sqrt{\frac{\varkappa'}{\varkappa}}v;\\ &f(u),\,g(v)\to F(U)=\sqrt{\frac{\varkappa'}{\varkappa}}f(u),\,G(V)=\sqrt{\frac{\varkappa}{\varkappa'}}g(v), \end{aligned} \tag{97}$$

in their expressions (2). Then

$$\alpha_{\omega'\omega}=A_{\Omega'\Omega},\quad \beta^*_{\omega'\omega}=B^*_{\Omega'\Omega}, \tag{98}$$

and the diagonal elements $A_{\Omega\Omega}$ with $\Omega=\Omega'=\sqrt{\omega\omega'}$ are involved in the trace (96).

For the trajectories in the Minkowsky plane on the left from their tangent line $X^\alpha(\tau')$ at zero point the coordinate $z^1=X^1(\tau')-x^1(\tau)\geqslant 0$. For these trajectories the $\mathrm{tr}\,\alpha$ can be transformed to the form [26]

$$\mathrm{tr}\,\alpha^{B,F}=\pm i\iint d\tau d\tau'\left\{\begin{array}{c}\dot x_\alpha(\tau)\dot X^\alpha(\tau')\\ 1\end{array}\right\}\varDelta_4^{LR}(z,\nu),\qquad z^\alpha=X^\alpha(\tau')-x^\alpha(\tau), \tag{99}$$

where the singular function $\Delta_4^{LR}(z,\nu)$ differs from the causal function $\Delta_4^f(z,\mu)$ by complex conjugation and the replacement $\mu \to i\nu$ (or by the replacement $z^2 \to -z^2,\ \mu \to \nu$):

$$\Delta_4^{LR}(z,\nu) = \frac{1}{4\pi}\delta(z^2) - \frac{\nu}{8\pi\sqrt{z^2}}\theta(z^2)H_1^{(2)}(\nu\sqrt{z^2}) + i\frac{\nu}{4\pi^2\sqrt{-z^2}}\theta(-z^2)K_1(\nu\sqrt{-z^2}). \qquad (100)$$

The expression obtained allows to interpret $\mathrm{tr}\,\alpha^{B,F}$ as a functional describing the interaction of two vector or scalar sources by means of exchange by vector or scalar quanta with spacelike momenta. At the same time one of the sources moves along the mirror's trajectory while another one moves along the tangent line to it at zero point. The last source can be considered as a probe or detector of excitation created by the accelerated mirror in vacuum.

As the detector moves with constant velocity β_0, its 2-velocity $\dot{X}^\alpha(\tau')$ does not depend on τ'. Consequently, $\dot{x}_\alpha(\tau)\dot{X}^\alpha(\tau') = -\gamma_*(\tau)$ is the relative Lorentz-factor defined by the relative velocity $\beta_*(\tau)$ of the mirror and detector:

$$\gamma_*(\tau) = \frac{1-\beta(\tau)\,\beta_0}{\sqrt{1-\beta^2(\tau)}\,\sqrt{1-\beta_0^2}} = \frac{1}{\sqrt{1-\beta_*^2(\tau)}}, \qquad \beta_*(\tau) = \frac{\beta(\tau)-\beta_0}{1-\beta(\tau)\,\beta_0}, \qquad (101)$$

and is the Lorentz-invariant quantity for each τ. Then

$$\mathrm{tr}\,\alpha^{B,F} = -i\int d\tau \left\{ \begin{array}{c} \gamma_*(\tau) \\ 1 \end{array} \right\} J(\tau,\nu), \qquad J(\tau,\nu) = \int d\tau'\, \Delta_4^{LR}(z(\tau,\tau'),\nu). \qquad (102)$$

It is seen from this representation that at $\theta \neq \infty$, when Lorentz- factor $\gamma_*(\tau)$ is confined on the whole trajectory, the both traces have the same qualitative behaviour when parameter $\nu \to 0$. It is clear that their infrared (logarithmic) singularities in this parameter are indebted to the behaviour of the integral $J(\tau,\nu)$ at $\tau \to \pm\infty$. For the trajectories with subluminal relative velocities β_{10}, β_{20} of the ends both $\mathrm{tr}\,\alpha^{B,F}$ have infrared singularities at $\nu = 0$. Besides, the singularities of $\mathrm{tr}\,\alpha^B$ differ from those of $\mathrm{tr}\,\alpha^F$ only by the values of the relative Lorentz-factor $\gamma_*(\tau)$ for initial and final ends of the trajectory, i.e. by the factors $1/\sqrt{1-\beta_{10}^2}$ and $1/\sqrt{1-\beta_{20}^2}$. Since the infrared singularities from the initial and final ends appear in $\mathrm{tr}\,\alpha^F$ with the factors

$$\frac{\sqrt{1-\beta_{10}^2}}{2\beta_{10}}, \qquad \frac{\sqrt{1-\beta_{20}^2}}{2|\beta_{20}|}, \qquad (103)$$

they disappear in $\mathrm{tr}\,\alpha^F$ for the trajectories with luminal velocities of the ends, $\beta_{10} = 1$, $\beta_{20} = -1$, but remain in $\mathrm{tr}\,\alpha^B$. The disappearance of singularities in $\mathrm{tr}\,\alpha^F$ for the such trajectories means that the function $J(\tau,\,\nu)$ is integrable in τ at $\tau \to \pm\infty$ even if $\nu = 0$. At the same time the function $\gamma_*(\tau)\,J(\tau,\nu)$ is integrable in this region only at $\nu \neq 0$.

The weakening of interaction of scalar charges with increasing their relative velocity, contrary to the constancy of interaction of electric charges, is connected with different geometrical structure of scalar and vector field sources $\rho(x)$ and $j^\alpha(x)$. They are given by (4) for pointlike charges moving along the trajectory $x^\alpha(\tau)$.

The charges of the scalar and vector field sources are defined by the space integrals of their charge densities $\rho(\mathbf{x},t)$ and $j^0(\mathbf{x},t)$, and for pointlike sources equal to

$$Q_0,\ Q_1 = \int d^3x\,\{\rho(\mathbf{x},t),\ j^0(\mathbf{x},t)\} = e\int d\tau\{1,\ \dot{x}^0(\tau)\}\,\delta(t-x^0(\tau)) = e\{\gamma^{-1}(t),\ 1\}, \qquad (104)$$

since $d\tau/dt' = \gamma^{-1}(t')$ if $t' = x^0(\tau)$. As is obvious, the charge for the pointlike source $T^{\alpha\beta}(x)$ of a tensor field with spin 2 increases as the particle's energy, $Q_2 = e\gamma(t)$.

As is seen from regularized representation

$$\mathrm{tr}\,\alpha^{B,F}|_{reg} = \frac{1}{2\pi}\int_0^\infty ds[\int_{-\infty}^\infty dx\,\{1,\,\sqrt{G'(x)}\}\,e^{-is(G(x)-x)} - \sqrt{\frac{\pi}{ibs}}],\quad s = \frac{\omega}{\varkappa}, \tag{105}$$

obtained in [26], the ultraviolet divergences in $\mathrm{tr}\,\alpha^{B,F}$ are removed by subtraction from the integrand of the first term its asymptotical expansion in s, as $s \to \infty$. The invariant variable $s = \omega/\varkappa = \sqrt{\omega\omega'/\varkappa\varkappa'} = b\rho/2w_0$ is proportional to momentum transfer ρ in units of proper acceleration w_0 of the mirror at the point of its tangency with detector. The subtracted term, being integrated over ρ up to large but finite ρ_{max},

$$\frac{1}{2\pi}\int_0^{s_{max}} ds\sqrt{\frac{\pi}{ibs}} = \frac{1}{2\pi}\sqrt{\frac{\pi\rho_{max}}{w_0}}(1-i), \tag{106}$$

is one and the same for Bose and Fermi cases and explicitly depends on acceleration.

When the space interval Δx between the mirror and detector becomes less than $\hbar/2\Delta p$, the uncontrolled momentum transfer between them becomes greater than Δp and leads to ultraviolet divergency in nonregularized $\mathrm{tr}\,\alpha^{B,F}$. As the mirror coordinate near the point of tangency with detector changes in time according to the law $x(t) = -w_0\,t^2/2$, the time interval τ necessary for the momentum transfer Δp is of the order of $\tau \sim 2\sqrt{\hbar/\Delta p\,w_0} = 2/\sqrt{w_0\rho_{max}}$ if one sets $\Delta p = \hbar\rho_{max}$. Then the subtracted term which regularizes the $\mathrm{tr}\,\alpha^{B,F}$ acquires the form

$$\frac{1}{2\pi}\sqrt{\frac{\pi\rho_{max}}{w_0}}(1-i) = \frac{1}{4\pi}\sqrt{\pi}\rho_{max}\,(1-i)\cdot\tau, \qquad \tau \sim 2/\sqrt{w_0\rho_{max}}. \tag{107}$$

As distinct from (20), this term has one and the same sign for Bose and Fermi cases. This can be understood as a consequence of positive momentum transfer from detector to mirror in both cases. The differences in meanings of $\rho_{max} \sim 1/\sqrt{2\varepsilon}$ and τ are more understandable.

Unlike $\Delta W_{1,0}$, describing the change of selfaction of a charges due to acceleration, the functionals $\mathrm{tr}\,\alpha^{B,F}$ describe the interaction of accelerated mirror with the probe executing uniform motion along the tangent to the mirror's trajectory at the point where mirror has acceleration w_0. This interaction is transmitted by the vector or scalar perturbations created by the mirror in the vacuum of Bose- or Fermi-field and carrying the spacelike momentum of the order of w_0. According to (100), at distances of the order of w_0^{-1} from the mirror, the field of these perturbations decreases exponentially in timelike directions and oscillates with damped amplitude in spacelike directions. It can be said that such a field moves together with the mirror and is its "proper field". Hence, the probe interacts with the mirror for a time of the order of w_0^{-1} while the charge all the time interacts with itself and feels the change of interaction over the all time of acceleration. Therefore, it is not surprising that the $-\mathrm{tr}\,\alpha^{B,F}$ coincide in essence with $\Delta W_{1,0}$ if in these latter one puts $\tau_2 - \tau_1 = 2\pi/w_0,\ e^2 = 1$. In other words, the $\mathrm{tr}\,\alpha^{B,F}$ are the mass shifts of the mirror's proper field multiplied by characteristic proper time of their formation.

7 Interaction with proper field of the accelerated mirror moving with subluminal velocity

The $\mathrm{tr}\,\alpha$ for the trajectory with subluminal velocities of the ends is an invariant function of the relative velocities β_{12}, β_{10}, β_{20} connected by the relation $\beta_{12} = (\beta_{10} - \beta_{20})/(1 - \beta_{10}\beta_{20})$. Let us consider the regularized $\mathrm{tr}\,\alpha^{B,F}$ for two important trajectories.

1. Quasihyperbolic trajectory, given by the formula (22), is time-reversed to itself. Its representation in u, v-variables is the following

$$u^{mir} = g(v) = v\,\mathrm{ch}\,\theta - \frac{\beta_1}{w_0}\mathrm{sh}\,\theta + \mathrm{sh}\,\theta\sqrt{(v - \frac{\beta_1^2}{w_0})^2 + a^2}, \quad a = \frac{\beta_1\sqrt{1-\beta_1^2}}{w_0}, \quad \beta_1 = \mathrm{th}\,\frac{\theta}{2}. \tag{108}$$

The initial β_1 and final $\beta_2 = -\beta_1$ velocities are subluminal.

Upon using this expression in the representation (2) and introducing the variable $x = v - \beta_1^2/w_0$ we obtain

$$\alpha^B_{\omega'\omega} = 2\sqrt{\frac{\omega'}{\omega}}\, e^{i\frac{\omega+\omega'}{w_0}\beta_1^2} \int_0^\infty dx\, \cos\left[(\omega' - \omega\,\mathrm{ch}\,\theta)x\right] e^{-i\omega\,\mathrm{sh}\,\theta\,\sqrt{x^2+a^2}}. \tag{109}$$

According to the formulae 9 and 15 of the section 2.5.25 in [27] this integral reduces to the modified Bessel and Hankel functions and we finally have

$$\alpha^B_{\omega'\omega} = 2ia\,\mathrm{sh}\,\theta\sqrt{\frac{\omega\omega'}{Q}}\, e^{i\frac{\omega+\omega'}{w_0}\beta_{10}^2}\, K_1(a\sqrt{Q}), \quad -\pi a\,\mathrm{sh}\,\theta\sqrt{\frac{\omega\omega'}{-Q}}\, e^{i\frac{\omega+\omega'}{w_0}\beta_{10}^2}\, H_1^{(2)}(a\sqrt{-Q}), \tag{110}$$

for $Q = \omega^2 + \omega'^2 - 2\,\omega\omega'\,\mathrm{ch}\,\theta \gtrless 0$. As usual, $\theta = \mathrm{Arth}\,\beta_{12}$ is the Lorentz-invariant parameter defined by the relative velocity of the ends.

The corresponding Bogoliubov coefficient for Fermi-case is more complicate:

$$\alpha^F_{\omega'\omega} = a\, e^{i\frac{\omega+\omega'}{w_0}\beta_{10}^2} \int_{-\infty}^{\infty} dt\, \sqrt{\mathrm{sh}^2 t + \mathrm{ch}^2\frac{\theta}{2}}\, \exp[ia((\omega' - \omega)\,\mathrm{ch}\frac{\theta}{2}\,\mathrm{sh} t - (\omega' + \omega)\,\mathrm{sh}\frac{\theta}{2}\,\mathrm{ch} t)]. \tag{111}$$

As the velocity of the mirror at the point $u = v = 0$ (and then the detector velocity) is equal to zero, $\beta_0 = 0$, the initial and final velocities β_1, β_2 can be considered as invariant relative velocities $\beta_1 = \beta_{10}$, $\beta_2 = \beta_{20}$ of the mirror and detector at $t = \mp\infty$.

According to the definition (105) we obtain

$$\mathrm{tr}\,\alpha^B|_{reg} = \frac{\mathrm{cth}\,\theta/2}{2\pi}\,[-\frac{\pi}{2} - i(\ln\frac{2}{\gamma\varepsilon} - 1)], \qquad \varepsilon = \nu/w_0, \tag{112}$$

$$\mathrm{tr}\,\alpha^F|_{reg} = \frac{1}{2\pi}\,\{\frac{1}{\mathrm{sh}\,\theta/2}\,[-\frac{\pi}{2} - i(\ln\frac{2}{\gamma\varepsilon} - 1)] + i[\mathrm{th}\,\frac{\theta}{2}\,\mathbf{B}(k) + \frac{\ln\,\mathrm{ch}\,\theta/2}{\mathrm{sh}\,\theta/2}]\}, \tag{113}$$

$$\mathbf{B}(k) = \int_0^{\pi/2} \frac{\cos^2\varphi\, d\varphi}{\sqrt{1 - k^2\sin^2\varphi}}, \quad k = \mathrm{th}\,\frac{\theta}{2}.$$

Here $\mathbf{B}(k)$ is one of the elliptic integrals [14].

In both $\mathrm{tr}\,\alpha^{B,F}$ the infrared singularities were removed by introducing the small parameter ε (the least momentum transfer in w_0 units), while the ultraviolet singularities were eliminated as was written above.

The function
$$R(\theta) = \mathrm{th}\,\frac{\theta}{2}\,\mathbf{B}(k) + \frac{\ln\,\mathrm{ch}\,\theta/2}{\mathrm{sh}\,\theta/2}$$
is equal to zero at $k=0$, grows almost linearly with k, reaches maximal value ≈ 1.28 at $k \approx 0.97$ and then decays rapidly to 1 as $k \to 1$.

2. The Airy's semiparabola with in-tangent line to inflection point is given by
$$\begin{aligned} \varkappa u^{\mathrm{mir}}(v) &= (1-b^2/c)\,\varkappa' v - b^3/3c^2, \qquad -\infty < v \leqslant v_0, \\ &= \varkappa' v + b\,\varkappa'^2 v^2 + (1/3)\,c\,\varkappa'^3 v^3, \qquad v_0 \leqslant v < \infty, \end{aligned} \tag{114}$$
where the inflection point $v_0 = -b/\varkappa' c$, $b > 0$, and $c > b^2$ due to the timelikeness of the trajectory. The initial velocity is subluminal, but the final one is luminal.

By using this trajectory in integral representations for Bogoliubov coefficients we find that
$$\alpha^B_{\omega'\omega} = \sqrt{\frac{s'/s}{\varkappa\varkappa'}}(cs)^{-1/3} e^{i(b/c)(s-s')-i(2/3)w^{3/2}}\,[\pi\,\mathrm{Ai}(z) - i(\pi\,\mathrm{Gi}(z) - \frac{1}{z})], \tag{115}$$

$$\alpha^F_{\omega'\omega} = \frac{(cs)^{-1/3}}{\sqrt{\varkappa\varkappa'(\alpha+1)}}\,e^{i(b/c)(s-s')-i(2/3)w^{3/2}}[\frac{i\sqrt{\alpha}}{z} + \frac{1}{\sqrt{w}}\int_0^\infty dt\,\sqrt{t^2+\alpha w}\,e^{-izt-it^3/3}]. \tag{116}$$
Here $\mathrm{Ai}(z)$ and $\mathrm{Gi}(z)$ are well known Airy and Scorer functions defined as in [28], and
$$z = (cs)^{-1/3}(s-s') - w, \quad w = (b/c)^2(cs)^{2/3}, \quad s = \omega/\varkappa, \quad s' = \omega'/\varkappa', \quad \alpha = c/b^2 - 1,$$
Parameter $\alpha = (1-\beta_{10})/2\beta_{10}$ is defined by the initial relative velocity β_{10} of the mirror and detector, $\beta_{20} = -1$.

At the finding $\mathrm{tr}\,\alpha^B$ the integral
$$\mathrm{tr}\,\alpha^B = \frac{1}{2\pi}\,\frac{3}{2}(\alpha+1)\int_0^\infty dw\,e^{-i\frac{2}{3}w^{3/2}}[\pi\mathrm{Ai}(-w) - i(\pi\mathrm{Gi}(-w) + \frac{1}{w})] \tag{117}$$
appears which diverges both on the lower and upper limits. The infrared divergency is remedied by introducing the nonzero lower limit $w_1 = (\varepsilon/2(\alpha+1)^2)^{2/3}$ where $\varepsilon = \nu/w_0 \ll 1$. To eliminate the ultraviolet divergency we subtract from the integrand the first term $\sqrt{\pi}e^{-i\pi/4}w^{-1/4}$ its asymptotical expansion for $w \to \infty$. Now it is possible to turn the integration contour on the angle $-\pi/3$ and, introducing the integration variable $t = e^{i\pi/3}w$, to brought the regularized integral to the form
$$\mathrm{tr}\,\alpha^B|_{reg} = \frac{1}{2\pi}\,\frac{3}{2}(\alpha+1)\{-\frac{\pi}{3} - i\int_{t_1}^\infty \frac{dt}{t}\,e^{-\frac{2}{3}t^{3/2}} + i\int_0^\infty dt\,\pi\mathrm{Gi}(t)e^{-\frac{2}{3}t^{3/2}}\}. \tag{118}$$
By the way we used the formulae
$$\mathrm{Ai}(e^{2\pi i/3}t) = \frac{1}{2}e^{i\pi/3}[\mathrm{Ai}(t) - i\mathrm{Bi}(t)], \quad \mathrm{Gi}(e^{2\pi i/3}t) = -e^{i\pi/3}\mathrm{Gi}(t) + \frac{1}{2}e^{-i\pi/6}[\mathrm{Ai}(t) + i\mathrm{Bi}(t)],$$
$$\int_0^\infty dt\,(\pi\mathrm{Bi}(t)e^{-\frac{2}{3}t^{3/2}} - \frac{\sqrt{\pi}}{t^{1/4}}) = 0. \tag{119}$$

The last integral in (118) is equal to $\frac{2}{3}+\frac{2}{9}\ln 2$. As a result we obtain finally

$$\mathrm{tr}\,\alpha^B|_{reg}=\frac{1}{2\pi}(\alpha+1)\{-\frac{\pi}{2}-i[\ln\frac{3(\alpha+1)^2}{\gamma\varepsilon}-1-\frac{1}{3}\ln 2]\},\quad \varepsilon=\nu/w_0, \tag{120}$$

The evaluation of $\mathrm{tr}\,\alpha^F$ follows by the similar way. Now the integral

$$\mathrm{tr}\,\alpha^F=\frac{1}{2\pi}\frac{3}{2}\sqrt{\alpha+1}\int_0^\infty dw\,e^{-i\frac{2}{3}w^{3/2}}[\frac{1}{\sqrt{w}}\int_0^\infty dt\,\sqrt{t^2+\alpha w}\,e^{iwt-it^3/3}-\frac{i}{w}\sqrt{\alpha}\,] \tag{121}$$

appears instead of the integral (117). The main terms of asymptotical expansions of the integrand for $w\to 0$ and $w\to\infty$ are identical to those of the integrand in (117) and differ from them only by extra multipliers $\sqrt{\alpha}$ and $\sqrt{\alpha+1}$ correspondingly. After elimination of the infrared and ultraviolet divergences and turning the integration contour on the angle $-\pi/3$ we obtain

$$\mathrm{tr}\,\alpha^F|_{reg}=\frac{1}{2\pi}\{\sqrt{\alpha(\alpha+1)}(-\frac{\pi}{2}-i\ln\frac{3(\alpha+1)^2}{\gamma\varepsilon})+i\sqrt{\alpha+1}\,J(\alpha)\}, \tag{122}$$

where

$$J(\alpha)=-3\int_0^\infty dx\,[\,e^{-\frac{2}{3}x^3}\int_0^\infty d\tau\,\sqrt{\tau^2+\alpha x^2}\,e^{x^2\tau-\tau^3/3}-\sqrt{\pi(1+\alpha)x}\,]. \tag{123}$$

S.L. Lebedev called author's attention to the fact that the integral $J(\alpha)$ can be reduced to elementary functions. Indeed, it can be shown that

$$J(\alpha)=1+\sqrt{\alpha}+\frac{\alpha-2}{3\sqrt{\alpha+1}}\ln\frac{\alpha+\sqrt{\alpha(\alpha+1)}}{1+\sqrt{\alpha+1}}+\frac{\sqrt{4+\alpha}}{3}\ln\frac{\sqrt{\alpha(4+\alpha)}-\alpha}{4+2\sqrt{4+\alpha}}. \tag{124}$$

The function $J(\alpha)$ is equal to $1-\frac{2}{3}\ln 2=0.5379\ldots$ at $\alpha=0$, attains minimal value ≈ 0.39 at $\alpha\approx 0.3$, and then grows and behaves as $(1+\frac{1}{3}\ln 2)\sqrt{\alpha}$ as $\alpha\to\infty$.

Note, that $\alpha^{B,F}_{\omega'\omega}$ depend on two dimensionless parameters b, c, but the traces $\mathrm{tr}\,\alpha^{B,F}$ depend only on their combination α, i.e. only on the subluminal relative velocity β_{10}.

Airy semiparabola with out-tangent line is time-reversed to the considered trajectory and can be obtained from it by the changes $v\rightleftarrows -u$, $\varkappa\rightleftarrows\varkappa'$. This leads to the change $s\rightleftarrows s'$ in the expressions for $\alpha^{B,F}_{\omega'\omega}$. The $\mathrm{tr}\,\alpha^{B,F}$ do not change at all, but it must be understood that the parameter α is now defined by the final (and negative) relative velocity β_{20} of the mirror and detector: $\alpha=-(1+\beta_{20})/2\beta_{20}>0$, while $\beta_{10}=1$.

The infrared logarithmic singularities of $\mathrm{tr}\,\alpha^{B,F}$ were regularized by nonzero momentum transfer $\nu\ll w_0$. Their coefficients are in accordance with general consideration of Section 6. These singularities disappear from $\mathrm{tr}\,\alpha^F|_{reg}$ at luminal velocities of the ends, and $\mathrm{tr}\,\alpha^F|_{reg}$ becomes pure imaginery positive. The positive sign of the $\mathrm{Im}\,\mathrm{tr}\,\alpha^F|_{reg}$ in this case can be explained by the large momentum transfer to the mirror during its contact with detector while the negative signs of $\mathrm{Im}\,\varDelta m_0$ and $\mathrm{Im}\,\varDelta m_1$ are connected with energy-momentum losses by a charge due to the change of self-interaction at acceleration.

We do not consider here the coefficients $\beta^{B,F*}_{\omega'\omega}$. They can be obtained from $\alpha^{B,F}_{\omega'\omega}$ by the changes $\omega\to-\omega$, $\sqrt{\omega}\to -i\sqrt{\omega}$, and division on i in Bose-case, see (2).

8 Conclusion

The symmetry being discussed was revealed itself in the coincidence of the bilinear in $\beta_{\omega'\omega}$ quantities, such as

$$|\beta_{\omega'\omega}|^2, \quad (\beta^+\beta)_{\omega\omega} = \int_0^\infty \frac{d\omega'}{2\pi}\,\beta^*_{\omega'\omega}\,\beta_{\omega'\omega}, \quad \bar{N} = \mathrm{tr}\,\beta^+\beta = \int_0^\infty \frac{d\omega}{2\pi}\,(\beta^+\beta)_{\omega\omega},$$

with the corresponding quantities describing the emission of vector (scalar) quanta by electric (scalar) charge in 3+1-space, see Introduction. Only likely transforming frequencies are involved in each summation entering in these quantities as well as in the equality $\omega = \omega''$ for the diagonal elements of the matrix $(\beta^+\beta)_{\omega\omega''} = \int_0^\infty \frac{d\omega'}{2\pi}\,\beta^*_{\omega'\omega}\beta_{\omega'\omega''}$. On the other hand, the definition of the trace of matrix $\alpha_{\omega'\omega}$ with differently transforming indexes ω, ω' required the Lorentz- invariant frequencies Ω, Ω', coinciding with ω, ω' in the proper system of the detector, moving along the tangent line to the mirror's trajectory at characteristic point. As a result, the $\mathrm{tr}\,\alpha$ becomes a functional of not only the mirror's trajectory but also the detector's one. This allows to consider the $\mathrm{tr}\,\alpha$ as experimentally measurable quantity.

The symmetry under discussion has been embodied in several exact mathematical relations between important physical quantities. The most important of them are, of course, the fundamental relations (11), (12) between the Bogoliubov coefficients for the processes induced by a mirror in 1+1-space and the current and charge densities for the processes induced by a charge in 3+1-space. Another one is the integral connection (16) between the propagator of a pair of massless particles, scattered in 1+1-space in opposite directions with frequencies ω, ω' (so that the pair has a mass $m = 2\sqrt{\omega\omega'}$), and the propagator of a single particle in 3+1-space. This relation provides the connection $\mathit{\Delta} W_{1,0} = e^2\,\mathit{\Delta} W^{B,F}$ between the self-action changes of a charge in 3+1-space and of a mirror in 1+1-space if $\mathrm{tr}\,\beta^+\beta \ll 1$.

The other relations with the symmetry embodied in are the spectral representations for the real parts of the self-action changes (32) and of the mass shifts (34),(38) of electric and scalar charges in quasihyperbolic motion. So, the mass shifts of a charges, the sources of the Bose-fields with spin 1 and 0 in 3+1-space, are represented by the spectral distributions of the heat capacity or the energy of Bose- and Fermi-gases of massless particles in 1+1-space. The spectral representations allow to consider the mass shift formation as the balance between the radiation and excitation of the proper energy at acceleration.

The symmetry between processes induced by the mirror in two-dimensional and by the charge in four-dimensional space-times predicts not only the value $e_0^2 = 1$ for the bare charge squared that corresponds to the bare fine structure constant $\alpha_0 = 1/4\pi$. It predicts also the appearance of scalar particles in ultra high-energy collisions in 3+1-space and the decreasing their interaction with scalar source with increasing of the energy.

It is very interesting that the bare fine structure constant has the purely geometrical origin, and, also, that its value is small: $\alpha_0 = 1/4\pi \ll 1$. The smallness of α_0 has the essential meaning for the quantum electrodynamics where it explains the smallness of α and justifies a priori the applicability of the perturbation theory.

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References

[1] S.W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[2] A.I. Nikishov, V.I. Ritus, Zh. Eksp. Teor. Fiz. **108**, 1121 (1995).

[3] V.I. Ritus, Zh. Eksp. Teor. Fiz. **110**, 526 (1996).

[4] B.S. DeWitt, Phys. Rep. C**19**, 295 (1975).

[5] R.M. Wald, Commun. Math. Phys. **45**, 9 (1975).

[6] V.I. Ritus, Zh. Eksp. Teor. Fiz. **114**, 46 (1998).

[7] G.'t Hooft, *Dimensional Reduction in Quantum Gravity*, Utrecht Preprint THU-93/26, gr-qc/9310006.

[8] L. Susskind, J. Math. Phys. **36**, 6377 (1995).

[9] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B**428**, 105 (1998).

[10] J. Maldacena, Adv. Theor. Phys. **2**, 231 (1998).

[11] J. Schwinger, *Particles, Sources and Fields*, v.1 (Addison-Wesley, Reading, 1970).

[12] A.P. Lightman, W.H. Press, R.H. Price, S.A. Teukolsky, *Problem Book in Relativity and Gravitation* (Princeton University Press, Princeton, New Jersey, 1975).

[13] V.I. Ritus, Zh. Eksp. Teor. Fiz. **116**, 1523 (1999); hep-th/9912004.

[14] H. Bateman and A. Erdelyi, *Higher Transcendental Functions*, vv. 1,3 (Nauka, Moscow, 1967).

[15] V.I. Ritus, Zh. Eksp. Teor. Fiz. **75**, 1560 (1978).

[16] V.I. Ritus, Zh. Eksp. Teor. Fiz. **80**, 1288 (1981).

[17] G.M. Fichtenholz, *Course in Differential and Integral Calculus* (Nauka, Moscow, 1969).

[18] P.C.W. Davies, J. Phys. A: Math. Gen. **8**, 609 (1975).

[19] W.G. Unruh, Phys. Rev. D**14**, 870 (1976).

[20] L.D. Landau, E.M. Lifshitz, *Statistical Physics, Part 1* (Nauka, Moscow, 1976).

[21] M. Gell-Mann, F.E. Low, Phys. Rev. **95**, 1300 (1954).

[22] L.D. Landau, in *Niels Bohr and development of physics* (Ed. W. Pauli. L.: Pergamon press, 1955).

[23] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, *Quantum Electrodynamics* (Nauka, Moscow, 1989).

[24] L.D. Landau and E.M. Lifshitz, *Field Theory* (Nauka, Moscow, 1988).

[25] N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge Univ. Press, Cambridge, 1982).

[26] V.I. Ritus, Zh. Eksp. Teor. Fiz. **124**, 14 (2003).

[27] A.P. Prudnikov, Yu.A. Brychkov, O.I. Marichev, *Integrals and Series, Elementary Functions* (Nauka, Moscow, 1981).

[28] M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1964).